Relativistic Vlasov-Uehling-Uhlenbeck model

Zhen Zhang[1], Taesoo Song[2] and Che-Ming Ko[1]

[1] *clotron Institute and Department of Physics and Astronomy, Texas A&M University*

[2] Frankfurt Institute for Advanced Studies and Institute for Theoretical Physics, Johann Wolfgang Goethe Universität,

Transport 2017, FRIB-MSU

Introduction

- RVUU: 'Relativistic Vlasov-Uehling-Uhlenbeck' transport model.

  C. M. Ko, Q. Li, and R.-C. Wang, Phys. Rev. Lett. 59, 1084 (1987)

  C. M. Ko and Q. Li, Phys. Rev. C 37, 2270 (1988)

  C. M. Ko and G.-Q. Li, J. Phys. G 22, 1673 (1996)

- Based on the relativistic VUU equation derived from the nonlinear relativistic mean field model.
- Energy range : $0.05 \sim 2$ GeV
- Particles included: $\Delta, \pi, K, \bar{K}, \Lambda, \Sigma, \Xi \cdots$
- Including threshold effects

  T. Song and C. M. Ko, Phys. Rev. 91, 014901 (2015)

- Including pion s-wave and p-wave potentials.

  Z. Zhang and C. M. Ko, arXiv:1701.06682

Nonlinear Relativistic mean-field model

Lagrangian: B. Liu *et al.*, Phys. Rev. C 65, 045201 (2002).

$$\begin{aligned}\mathcal{L} &= \bar{N}[\gamma_\mu(i\partial^\mu - g_\omega\omega^\mu - g_\rho\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu) - (m_N - g_\sigma\sigma)]N \\ &+\frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{a}{3}\sigma^3 - \frac{b}{4}\sigma^4 \\ &-\frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ &+\frac{1}{2}(\partial_\mu\boldsymbol{\delta}\partial^\mu\boldsymbol{\delta} - m_\delta^2\boldsymbol{\delta}\cdot\boldsymbol{\delta}) \\ &-\frac{1}{4}\boldsymbol{R}_{\mu\nu}\cdot\boldsymbol{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu\end{aligned}$$

with $\Omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, $\boldsymbol{R}_{\mu\nu} = \partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu$.

- N: nucleon
- σ: isoscalar scalar meson with $m_\sigma = 550\,\mathrm{MeV}$
- ω^μ: isocalar vector meson with $m_\omega = 782\,\mathrm{MeV}$
- $\boldsymbol{\delta}$: isovector scalar meson with $m_\delta = 983\,\mathrm{MeV}$
- $\boldsymbol{\rho}^\mu$: isovector vector meson with $m_\rho = 769\,\mathrm{MeV}$

Mean-field approximation

Nucleon field equation

$$[\gamma_\mu(i\partial^\mu - g_\omega \omega^\mu - g_\rho \tau_3 \rho_3^\mu) - (m_N - g_\sigma \sigma - g_\delta \tau_3 \delta_3)]N = 0$$

becomes the field equation of non-interacting nucleons with the effective mass m_N^* and the kinetic energy-momentum $p_i^{\mu *}$,

$$\begin{aligned} m_p^* &= m - g_\sigma \sigma - g_\delta \delta_3 \\ m_n^* &= m - g_\sigma \sigma + g_\delta \delta_3 \\ p_p^{\mu *} &= p^\mu - g_\omega \omega^\mu - g_\rho \rho_3^\mu \\ p_n^{\mu *} &= p^\mu - g_\omega \omega^\mu + g_\rho \rho_3^\mu \end{aligned}$$

Mesons field equation

$$\begin{aligned} m_\sigma^2 \sigma + a\sigma^2 + b\sigma^3 &= g_\sigma(\phi_p + \phi_n) \\ m_\delta^2 \delta_3 &= g_\delta(\phi_p - \phi_n) \\ m_\omega^2 \omega^\mu &= g_\omega(j_p^\mu + j_n^\mu) \\ m_\rho^2 \rho_3^\mu &= g_\rho(j_p^\mu - j_n^\mu) \end{aligned}$$

Meson fields are expressed in terms of nucleon scalar and current density

$$\begin{aligned} \phi_i &= \int \frac{d^3\boldsymbol{p}_i}{(2\pi)^3} \frac{m_i^*}{E_i^*} f(\boldsymbol{p}_i) \\ j_i^\mu &= \int \frac{d^3\boldsymbol{p}_i}{(2\pi)^3} \frac{p_i^{\mu *}}{E_i^*} f_i(\boldsymbol{p}_i) \end{aligned}$$

with $E_i^* = \sqrt{m_i^* + p_i^*}$.

RVUU eqaution for nucleons

$$\frac{\partial}{\partial t} f + \boldsymbol{v} \cdot \boldsymbol{\nabla}_r f - \boldsymbol{\nabla}_r H \cdot \boldsymbol{\nabla}_p f = \mathcal{C}$$

C.M. Ko, Nucl. Phys. **A495**, 321 (1989)

- Mean field potential $H = \sqrt{M^* + p^{*2}} + g_\omega \omega^0 \mp g_\rho \rho_3^0$
- Collisional integral $\mathcal{C}$ includes:
  elastic scattering: $NN \to NN, N\Delta \to N\Delta$
  inelastic scattering: $NN \to N\Delta, N\Delta \to NN$.
- Test particle method: C. Y. Wong, Phys. Rev. C 25, 1460 (1982)

$$f(\boldsymbol{r}, \boldsymbol{p}; t) = \frac{1}{N_{\mathrm{TP}}} \sum_i^{AN_{\mathrm{TP}}} \delta\left[\boldsymbol{r} - \boldsymbol{r}_i(t)\right] \delta\left[\boldsymbol{p} - \boldsymbol{p}_i(t)\right]$$

All test particles are treated as point particle.

RVUU equations for Δ

Similar to RVUU equations for nucleons but with

$$
\begin{aligned}
m^*_{\Delta^{++}} &= m_\Delta - g_\sigma \sigma - g_\delta \delta_3 & p^\mu_{\Delta^{++}} &= p^{\mu *} + g_\omega \omega^\mu + g_\rho \rho_3^\mu \\
m^*_{\Delta^{+}} &= m_\Delta - g_\sigma \sigma + \frac{1}{3} g_\delta \delta_3 & p^\mu_{\Delta^{+}} &= p^{\mu *} + g_\omega \omega^\mu + \frac{1}{3} g_\rho \rho_3^\mu \\
m^*_{\Delta^{0}} &= m_\Delta - g_\sigma \sigma - \frac{1}{3} g_\delta \delta_3 & p^\mu_{\Delta^{0}} &= p^{\mu *} + g_\omega \omega^\mu - \frac{1}{3} g_\rho \rho_3^\mu \\
m^*_{\Delta^{-}} &= m_\Delta - g_\sigma \sigma + g_\delta \delta_3 & p^\mu_{\Delta^{-}} &= p^{\mu *} + g_\omega \omega^\mu - g_\rho \rho_3^\mu
\end{aligned}
$$

which are determined according to their isospin structures in terms of those of nulceons and pions

$$
|\Delta^{++}\rangle = |p\rangle|\pi^+\rangle \qquad |\Delta^{+}\rangle = \sqrt{\frac{2}{3}}|p\rangle|\pi^0\rangle + \sqrt{\frac{1}{3}}|n\rangle|\pi^+\rangle
$$

$$
|\Delta^{-}\rangle = |n\rangle|\pi^-\rangle \qquad |\Delta^{0}\rangle = \sqrt{\frac{1}{3}}|p\rangle|\pi^-\rangle + \sqrt{\frac{2}{3}}|n\rangle|\pi^0\rangle
$$

Equations of motion

- Baryons obey the classical equations of motion:

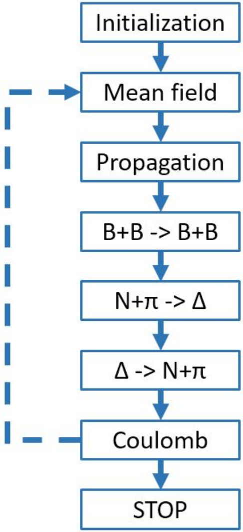
$$\begin{aligned} \dot{\boldsymbol{r}} &= \frac{\boldsymbol{p}^*}{E^*}, \\ \dot{\boldsymbol{p}} &= -\boldsymbol{\nabla} E, \end{aligned}$$

- Pions' equations of motion are similar

$$\begin{aligned} \dot{\boldsymbol{r}} &= \frac{d\omega}{d\boldsymbol{k}}, \\ \dot{\boldsymbol{k}} &= -\boldsymbol{\nabla} \omega. \end{aligned}$$

Optionally, the pion s-wave and p-wave potential can be included.

Structure



Initialization
Mean field
Propagation
B+B -> B+B
N+π -> Δ
Δ -> N+π
Coulomb
STOP

initialization

- Positions of nucleons in each nucleus are distributed according to
  - Wood-Saxon form

$$\rho(\boldsymbol{r}) = \frac{1}{1 + \exp[(r - c)/a]}$$

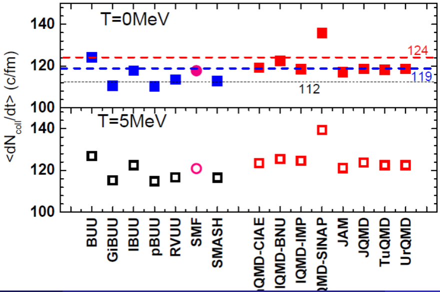
    Default valuse: $a = 0.535$ fm and $c = 1.2A^{1/3}$.
  - RMF calculation using the same parameter set.
- Momenta are initilized according to Fermi gas distribution determined by local density.

Bertsch's method: G. F. Bertsch and S. Das Gupta, Phys. Rep. 160, 189 (1988)

- The distance between two particles Δr should be less than $\sqrt{b^2+\Delta t^2}$, with $b=\sqrt{\sigma/\pi}$.
- In the c.m. frame

$$\sqrt{(\Delta \boldsymbol{r})^2+\left|\frac{\Delta \boldsymbol{r}\cdot\boldsymbol{p}}{p}\right|^2}<b; \quad \left|\frac{\Delta \boldsymbol{r}\cdot\boldsymbol{p}}{p}\right|<\left(\frac{p}{\sqrt{p^2+m_1^2}}+\frac{p}{\sqrt{p^2+m_2^2}}\right)\Delta t/2$$

- After removing spurious collision, the RVUU code gives very resonable collision number.



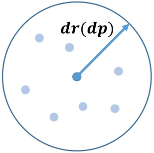
Pauli Blocking

For an emitted particle $(\boldsymbol{r}, \boldsymbol{p})$,

- Count n test particles of the same isospin state in the sphere of radius $dr(dp)$ around $\boldsymbol{r}(\boldsymbol{p})$.
- $f = \frac{n}{V N_{\mathrm{TP}}}$, with $V = \frac{4\pi}{3} dr^3 \frac{4\pi}{3} dp^3$
- For reaction $1 + 2 \to 3 + 4$, blocking probality is $1 - (1 - f_3)(1 - f_4)$

Default vaules of dr and dp :

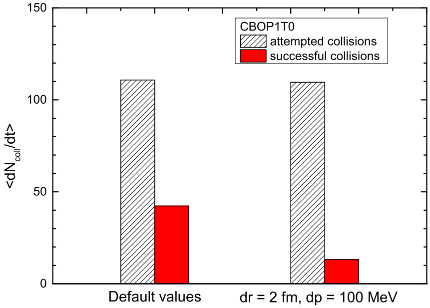
$$\begin{aligned} dr &= [3/(4\pi\rho_0)]^{1/3} \approx 1.14\mathrm{fm}, \\ dp &= [6\pi^2\rho_0/(2s+1)]^{1/3} \approx 331\mathrm{MeV}, \end{aligned}$$



with s being the spin degeneracy. $V = h^3/(2s+1)$.

Pauli Blocking

- For CBOP1T0, $dp = 331$ MeV is too large. (Fermi momentum is about 263 MeV)
- Using a smaller dp can improve the Pauli Blocking in RVUU:



Electromagetic fields

- The electric and magnetic fields acting on a charged particle i are given by

$$\boldsymbol{E}(\boldsymbol{r_i}) = \frac{e}{4\pi}\sum_{j\neq i} q_j \frac{\boldsymbol{r}_{ij}}{r_{ij}^3}$$

$$\boldsymbol{B}(\boldsymbol{r}_i) = \frac{e}{4\pi}\sum_{j\neq i} q_j \frac{\boldsymbol{\beta}_j \times \boldsymbol{r}_{ij}}{r_{ij}^3},$$

  where $\boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j$, $\boldsymbol{\beta}_j = \boldsymbol{p}_j^*/E_j^*$, and q_j is the electric charge of particle j in units of e. The sum is over all charged particles in one event.

- The momentum of particle i is changed due to the electric and magnetic fields by

$$\Delta \boldsymbol{p}_i = (\boldsymbol{E} + \boldsymbol{\beta}_i \times \boldsymbol{B}) q_i \Delta t$$

Hw Da2Pa

- Baryon-baryon elastic scattering

  G. F. Bertsch and S. Das Gupta, Phys. Rep. 160, 189 (1988)

  - total cross section

$$\sigma^{\rm elastic}_{BB\to BB}\,({\rm mb}) = \begin{cases} 55, & \sqrt{s} < 1.8993\text{GeV} \\ 20 + \frac{35}{1+100(\sqrt{s}-1.8993)}, & \sqrt{s} \geq 1.8993\text{GeV} \end{cases}$$

  - differential cross section

$$\frac{d\sigma^{\rm elastic}_{BB\to BB}}{dt} \sim \exp\left[\frac{6\left\{3.65(\sqrt{s}-1.866)\right\}^6}{1+\left\{3.65(\sqrt{s}-1.866)\right\}^6}t\right]$$

  with $t = -2p^2(1-\cos\theta)$.

- $N + N \to N + \Delta$ cross section is from one-boson exchange model.

  S. Huber and J. Aichelin, Nucl. Phys. A 573, 587 (1994)

Hw Da2Pa

- The Δ mass is sampled according to the function

$$P(m) = \frac{p_f m \Gamma_{\mathrm{tot}}(m)}{(m^2 - m_0^2)^2 + m_0^2 \Gamma_{\mathrm{tot}}^2(m)},$$

- $N'' + \Delta \to N + N'$ cross section is given by

$$\begin{aligned} \sigma(N''\Delta \to NN') &= \frac{m}{8m_0^2} \frac{1}{1+\delta_{NN'}} \frac{p_i^2}{p_f} \sigma(NN' \to N''\Delta) \\ &\times \left[\int_{m_{\mathrm{min}}}^{m_{\mathrm{max}}} \frac{dm}{2\pi} P(m) \right]^{-1}, \end{aligned}$$

where p_i and p_f is the nucleon kinetic momentum in the frame of $\boldsymbol{p}_N + \boldsymbol{p}_{N'} = 0$, and $\boldsymbol{p}_{N''} + \boldsymbol{p}_\Delta = 0$,respectively.

P. Danielewicz and G.F. Bertsch, Nucl. Phys. A533, 712 (1991)

B.A. Li and C.M. Ko, Phys. Rev. C 52, 2037 (1995)

Hw Da2Pa

- Δ decay width: B.A. Li and C.M. Ko, Phys. Rev. C 52, 2037 (1995)

$$\Gamma(q) = g \frac{0.47}{1 + 0.6(q/m_\pi)^2} \frac{q^3}{m_\pi^2},$$

where q is the momentum of emitted pion in the Δ rest frame.
The isospin factor g:

$$\begin{aligned} g = 1, &\qquad \text{for } \Delta^- \to n + \pi^-,\ \ \Delta^{++} \to p + \pi^+ \\ g = 2/3, &\qquad \text{for } \Delta^0 \to n + \pi^0,\ \ \Delta^+ \to p + \pi^0 \\ g = 1/3, &\qquad \text{for } \Delta^0 \to p + \pi^-,\ \ \Delta^+ \to n + \pi^+ \end{aligned}$$

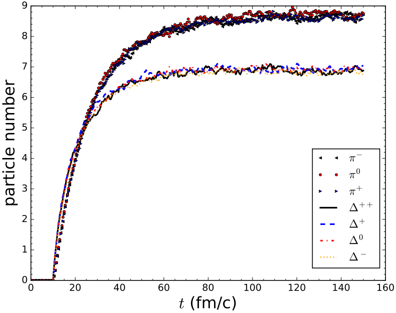
- $N\pi \to \Delta$ cross section:

$$\sigma = \frac{8\pi}{k^2} \frac{m_0^2 \Gamma \Gamma_{\text{tot}}}{(m_\Delta^2 - m_0^2)^2 + m_0^2 \Gamma_{\text{tot}}^2}$$

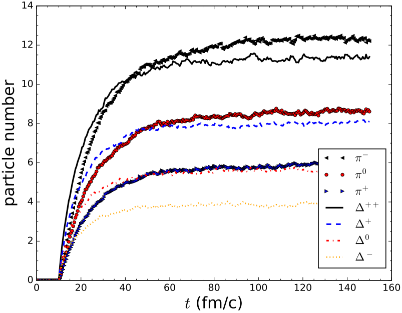
with Γ (Γ_{tot})) being the partial (total) Δ width.

Results for Da2Pa

$\delta = 0$



$\delta = 0.2$



Thank you!